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Analogue Filters Design
And Simulation.

4th order Butterworth response

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Abstract.

In this paper there is explained how to calculate a 4\textsuperscript{th} order low-pass Butterworth response by using a Sallen and key filter.

There are calculations on how to design the 4\textsuperscript{th} order Butterworth response, there is also simulations of the filter, using the program Tina.
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**Introduction.**

Active filters are used many places, for example in the telecommunication, in the audio frequency range (0 kHz to 20 kHz), and for modems to the internet connection, and for many more things.

There are many types of active filter response, some of them are Butterworth, Bessel, and Chebyshev, and the filter response can be used in Low-pass, High-pass, Band-pass and Band-stop filter setups.

Butterworth response is also named the maximally flat filter, because it is maximally flat in the pass-band response, the Butterworth response is perhaps the mote used type of filter response. The Chebyshev can has a ripple in the pass-band, this ripple can be from 0.01 dB and up to 3dB ripple, in return for the ripple in the pass-band the chebyshev response has very good cutoff at the edge of the pass-band as it is showed in figure 2 [4].

Figure 1 shows the poles in a 2nd order Butterworth, Bessel, and Chebyshev filters, there are no zeroes in this filter types.

The ideal Low-pas filter is showed in figure 3.
The polynomials for a 2\textsuperscript{nd} and 4\textsuperscript{th} order Butterworth filter [1].

- \(2^{nd} = \left(s^2 + 1,414s + 1\right)\)
- \(4^{th} = \left(s^2 + 0,765s + 1\right) \ast \left(s^2 + 1,848s + 1\right)\)

To make a 4\textsuperscript{th} order Butterworth filter there can be used tow 2\textsuperscript{nd} order sallen and key filters in series, figure 4 shows a 2\textsuperscript{nd} order sallen and key filter.

If the polynomials there are showed above, R1 and R2 shell have the same value and so shall C1 and C2, RA and RB determining the gain in the circuit.

![Figure 4](image_url)

**Specification for the assignment.**

- Fourth-order Butterworth response.
- Low-pass filter.
- Using modified Sallen-Key circuits.
- Cut-off frequency of 8 KHz.
Theoretical design of the 4th order Butterworth filter using Sallen and Key.

Circuit A.
C1 = C2 and are chosen to 2.2nF

\[ f_c = \frac{1}{2\pi R C} \Rightarrow R = \frac{1}{2\pi f_c C} \Rightarrow C = \frac{1}{2\pi f_c R} \]

\[ R = \frac{1}{2\pi f_c C} \Rightarrow \frac{1}{2\pi \times 8K \times 2,2n} = 9,04K\Omega \approx 9,2K\Omega_{E12} \]

R1 = R2 = 9.2KΩ

The gain in circuit A.:

\[ A_v O = 3 - 2K \Rightarrow 2K = 3 - A_v O = 0.765 \]

\[ A_v O = 3 - 2K \Rightarrow 3 - 0.765 = 2.235 \]

RA chosen to be 10KΩ

\[ A_v O = 1 + \frac{RA}{RB} \Rightarrow A_v O - 1 = \frac{RA}{RB} \Rightarrow \frac{RA}{(A_v O - 1)} = RB \]

\[ RB = \frac{RA}{A_v O - 1} \Rightarrow \frac{10K}{(2,235 - 1)} = 8,097K\Omega \approx 8,2K\Omega_{E12} \]
Circuit B.

R1, R2, C1 and C2 is the same as in circuit A, the only difference between circuit A and B is the gain.

\[ A_v O = 3 - 2K \Rightarrow 2K = 3 - A_v O = 1.848 \]
\[ A_v O = 3 - 2K \Rightarrow 3 - 1.848 = 1.152 \]

RA chosen to be 10KΩ again.

\[ RB = \frac{RA}{(A_v O - 1)} \Rightarrow 10K \approx 65,789KΩ ≈ 68KΩ_{12} \]

Total gain for the 4th order Butterworth filter circuit.

\[ A_v O = \text{gain}_A \ast \text{gain}_B \Rightarrow 2,235 \ast 1,152 = 2,575 \]
\[ dB = 20 \ast \log(A_v O) \Rightarrow 20 \ast \log(2.575) = 8.21dB \]
\[ dB = 20 \ast \log \left( \left(1 + \frac{R3}{R4}\right) \ast \left(1 + \frac{R7}{R8}\right) \right) \Rightarrow 20 \ast \log \left( \left(1 + \frac{10K}{8,09K}\right) \ast \left(1 + \frac{10K}{65,79K}\right) \right) = 8,218dB \]
\[ -3db = dB - 3 \Rightarrow 8,218 - 3 = 5,218dB \]

Circuit schematic.

Figure 6
Simulation of 4th order Butterworth filter in Tina.

As is it showed in figure 7 the -3dB point is at 8KHz as it should, the max gain is at 8.22db and has a gain of 2.58 times.

\[ Au = \log_{10}(\frac{20}{8.22}) \Rightarrow 10^{\log_{10}(\frac{20}{8.22})} = 2.58 \]

Figure 8 shows the phase of the 4th order Butterworth response as it is on -180 degrees, because there is a -45 degrees for each pole in the Butterworth response.
Figure 9 shows the gain between the signal In and the signal Out, and the gain is 2.6.

Input signal: 1KHz / 2V<sub>peak-peak</sub> sine wave.

\[
A_u = \frac{U_{out\ peak-peak}}{U_{in\ peak-peak}} \Rightarrow \frac{5.2}{2} = 2.6
\]
Signal In : 500Hz / 2V_{pek-pek} Square wave.

In figure 10 the output signal is similar to the input signal on 500Hz because the frequency is much lower than the -3dB point at 8KHz and the harmonic frequency is getting through the filter.

In figure 11 the input signal is a square wave on 8KHz and the harmonic frequency to 8KHz square wave signal is not getting through the filter, and therefore is the signal out of the filter is a sine wave on 8KHz.
Poles in the Butterworth filter using Sallen and key.

In a 4th order Butterworth filter is there no zeros only poles and there is as many poles as
the filter order.

2nd order filter = 2 poles

4th order filter = 4 poles

Then the transfer function has to be found, and it can be found by using Tina, and use it
in MathCAD to calculate the value of the imaginary and the real part.

The result from MathCAD is showed below

\[
s = \begin{pmatrix}
-1.921 \times 10^4 + 4.647\times 10^4 \\
-1.921 \times 10^4 - 4.647i \times 10^4 \\
-4.646 \times 10^4 + 1.923i \times 10^4 \\
-4.646 \times 10^4 - 1.923i \times 10^4
\end{pmatrix}
\]

Re = −19.21K + 46.47K

Im = −19.21K − 46.47K

Re = −46.46K + 19.23K

Im = −46.46K − 19.23K

In Tina a visual plot of the zeros can be simulated by using an ideal opamp figure 12.
Conclusion.

The calculations of the resistors R1, R2 and the capacitors C1, C2 is the same in circuit A and B, the only difference is the resistors RA and RB, they decide the gain, in the circuit.

To see if the calculations for the 4th order filter are right, the 8KHz have to be at -3dB point, this is indicate by the frequency response in figure 7 on page 8.

The gain in the filter is shown in figure 9 on page 9, it show a gain on 2.6, the calculation on page 7 show a gain of 2.57 this to calculations is almost the same, the calculation one at page 9 is calculated from a value there is read from figure 9 and is not as precise as the calculation on page 7.

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